Analytical Model for the Radial Strength and Collapse of the Bicycle Wheel

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1 INTRODUCTION

The bicycle wheel is a versatile engineering structure which serves many functions including supporting radial and lateral loads, transmitting propulsive torque, and sustaining braking loads. When a wheel is overloaded radially it typically buckles laterally (or "tacos") which renders the wheel unridable. It is both of theoretical interest and practical importance to develop a theory for the strength—and failure—of the wheel. We analyze the failure mechanisms of tension-spoked wheels using a combination of computational and theoretical approaches.

There are two primary failures which both lead to wheel collapse: loss of spoke tension, and lateral buckling of the rim. Designing against either failure mode presents a conflict because increasing spoke tension prevents spokes from going slack, but it also decreases the lateral stiffness of the rim, which is under compression due to the spokes. By analyzing these two modes separately and then equating the critical loads, we develop an expression for the wheel strength which is independent of spoke tension.

2 BUCKLING OF SPOKED WHEELS

A bicycle wheel may buckle laterally—or "taco"—due to excessive spoke tension, lateral force, or radial force. The rim takes on a saddle shape like the one shown in Figure 1. A wheelbuilder may unintentionally taco a wheel by applying a modest lateral force to a wheel with very high spoke tension. In fact, Jobst Brandt in his book *The Bicycle Wheel* [1] advocated using this approach to determine the maximum allowable tension, and then reducing tension slightly for production.

Figure 1: A buckled—or "tacoed"—wheel

We have previously shown [2] that the spoke tension required to spontaneously buckle a wheel is given by

$$T_{cn} = \frac{2RK_n}{n_s(n^2 - R/l_s)} \tag{1}$$

where R is the rim radius, K_n is a lateral deflection mode stiffness which combines the bending and torsional stiffness of the rim and the lateral projection of the axial stiffness of the spokes, n_s is the number of spokes, $n \geq 2$ is the buckling mode number, and l_s is the spoke length. The minimum mode buckling tension

$T_c = \min\{T_{cn}\}$ over all possible n gives the critical buckling tension. The lateral stiffness of the wheel decreases continuously with increasing spoke tension and vanishes when $T = T_c$.

2.1 Reduced-order model for rim buckling

If the spokes are sufficiently tight, the rim will buckle laterally under radial load before the spokes near the load point lose tension. The load point approximately follows a circular arc of radius R (Figure 2 (b)). This is analogous to the model presented in Figure 2 (c) consisting of a rigid, hinged bar restrained by a rotational spring. By assuming that the buckled shape of the wheel is identical to the displacement field under a pure lateral load, it can be shown that the critical radial load for the wheel is given by $P_{c,rim} = K_{lat}R$. The lateral stiffness K_{lat} depends on spoke tension. A rough approximation for the rim buckling load is

$$P_{c,rim} = K_{lat}(T)R \approx K_{lat}^0(1 - T/T_c)R. \tag{2}$$

where K_{lat}^0 is the lateral stiffness at zero spoke tension. Calculations from non-linear finite-element simulations of wheels with laterally restrained spokes (which do not buckle when tension is lost) show excellent agreement with the simplified model.

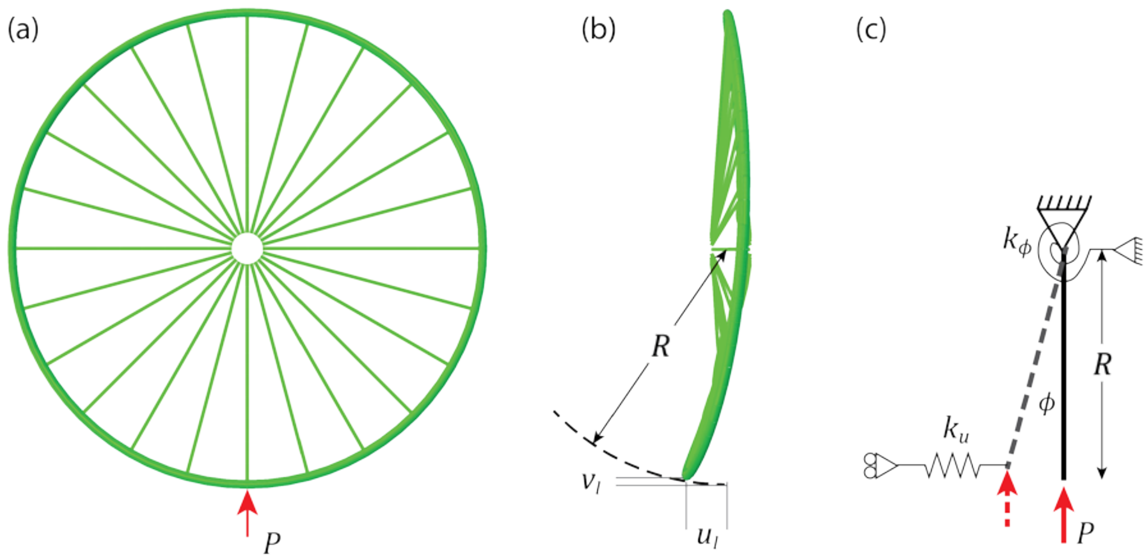

Figure 2: (a) Side view of a wheel loaded by a radial point load. (b) Front view of a wheel after rim buckling. (c) Schematic of the hinged-column model.

2.2 Extension of the model to realistic wheels

In real wheels, the spokes underneath the hub may lose tension and buckle before the rim reaches its critical load. The radial load to cause the bottommost spoke to buckle depends linearly on the spoke tension as follows:

$$P_{c,sp} = \left(\frac{T l_s}{EA}\right)\frac{K_{rad}}{\cos\alpha} \tag{3}$$

where T is the spoke tension, l_s and EA are the spoke length and axial stiffness, K_{rad} is the radial stiffness of the wheel, and α is roughly the inclination angle between a spoke and the wheel plane, defined in [2]. Equating Eqn. (2) and Eqn. (3) and solving for T gives the spoke tension for which spoke buckling and rim buckling occur simultaneously. Plugging back into Eqn. (2) and noting that $l_s \approx R$ and $\cos\alpha \approx 1$ gives the buckling strength of the wheel, independent of spoke tension.

$$P_{c,wheel} = K_{lat}^0 R \left(\frac{1}{1 + \frac{EA}{T_c}\frac{K_{lat}^0}{K_{rad}}} \right) \tag{4}$$

Our computational studies indicate that the opposing effects of high spoke tension—higher spoke buckling resistance but lower lateral stiffness—roughly cancel out. Therefore Eqn. (4) gives an estimate of the expected strength of the wheel at any tension.

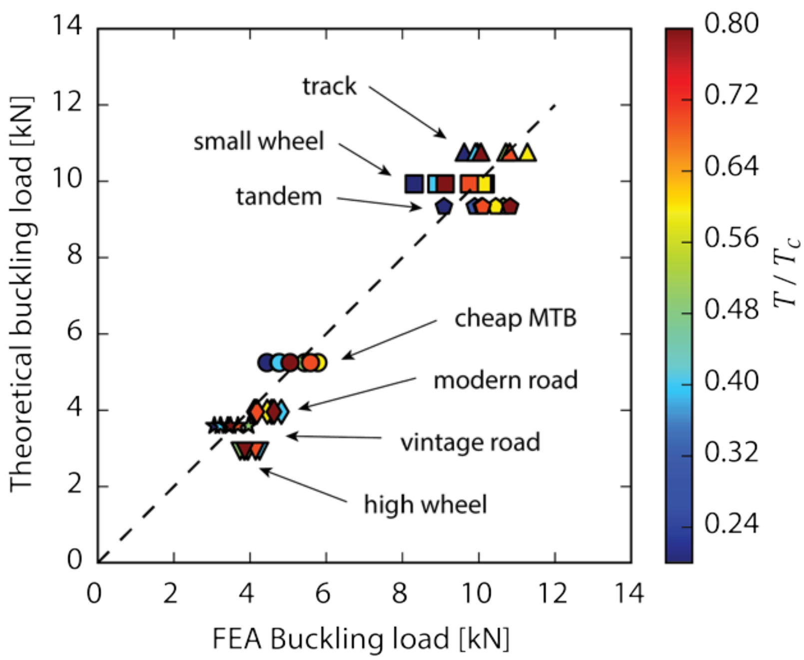

Figure 3: Comparison of Eqn. (4) and results from non-linear finite-element simulations of realistic wheels.

3 CONCLUSIONS

The strength of the bicycle wheel is primarily dictated by its lateral stiffness, as wheelbuilders have long assumed from intuition and experience. We have derived a very simple equation which predicts wheel strength over a wide range which accounts for the competing effects of spoke buckling and rim buckling. Our theory could be used to establish safety standards for wheel design and to guide the design of optimal wheels.

REFERENCES

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